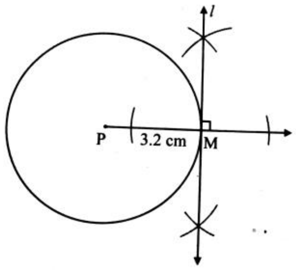
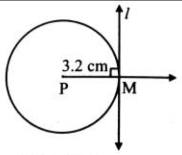
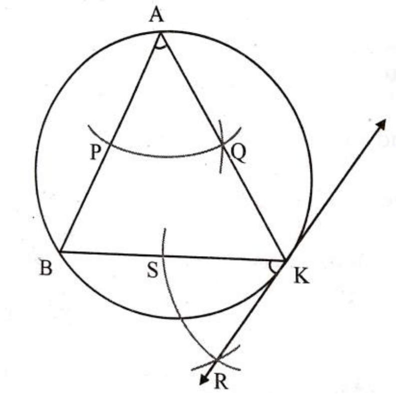
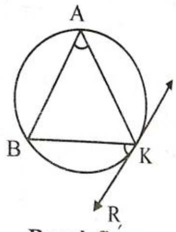
MKCL Creating a Knowledge Lit World www.mkcl.org	**Subject – Geometry** **Std. – 10^{th} EM/Semi**	**Set – I** **Model Answer**

Q.1.A	**Choose the correct alternative.**
1)	A
2)	A
3)	A
4)	B
Q.1.B	**Solve any TWO of the following:**
1)	**Rough Figure** **Steps of Construction:** i) With centre P, draw a circle of radius 3.2 cm. ii) Take any point M on the circle and draw ray PM. iii) Draw line *l* ⊥ ray PM at point M. Line *l* is the required tangent to the circle at point M.
2)	seg AD ∥ seg BC and seg BD is their transversal ....[Given] ∴ ∠DBC ≅ ∠BDA ....[Alternate angles] ∴ ∠PBC ≅ ∠PDA ....(i) [D – P – B] In ΔPBC and ΔPDA, ∠PBC ≅ ∠PDA ....[From (i) ∠BPC ≅ ∠DPA ....[Vertically opposite angles] ∴ ΔPBC ~ ΔPDA ....[AA test of similarity] ∴ $\frac{BP}{PD} = \frac{PC}{AP}$ ....[Corresponding sides of similar triangles] ∴ $\frac{\mathbf{AP}}{\mathbf{PD}} = \frac{\mathbf{PC}}{\mathbf{BP}}$ ....[By alternendo]
3)	Slope of line AB = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-1)}{0-(1)} = \frac{1+1}{0+1} = 2$ Slope of line BC = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{1-0} = 2$ ∴ The slopes of lines AB and BC are equal. ∴ line AB ∥ line BC Also, point B is common to both the lines. ∴ Both lines are the same. ∴ **Points A, B and C are collinear.**
Q.2A	**Complete the any TWO of the following activities:**
1)	In ΔABC, ray BD bisects ∠B. ...[Given] ∴ $\frac{AB}{BC} = \frac{AD}{DC}$ ...(i) [Angle bisector theorem] In ΔABC, DE ∥ BC ... [Given] ∴ $\frac{AE}{EB} = \frac{AD}{DC}$ ...(ii) [Basic proportionality theorem] ∴ $\frac{AB}{\boxed{BC}} = \frac{\boxed{AE}}{EB}$ ...[From (i) and (ii)]

2)	If PQ ∥ RS and P(1, −2), Q(5, 2), R(3, k) and S(k, −5). Complete the following activity to find value of k. ∴ Slope of PQ = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{\boxed{2} - \boxed{-2}}{\boxed{5} - \boxed{1}}$ $= \frac{\boxed{2+2}}{\boxed{5-1}} = \frac{\boxed{4}}{\boxed{4}} = \boxed{1}$ ...(i) Slope of RS = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{\boxed{-5} - \boxed{k}}{\boxed{k} - \boxed{3}}$ $= \frac{\boxed{-5-k}}{\boxed{k-3}}$ ...(ii) But slope of PQ = slope of RS (∵ $\boxed{\text{parallel}}$ lines have $\boxed{\text{equal}}$ slopes) ∴ $1 = \frac{-5-k}{k-3}$ ∴ $1(k-3) = -5-k$ ∴ $k-3 = -5-k$ ∴ $k+k = -5+3$ ∴ $2k = -2$ ∴ $k = \frac{-2}{2}$ ∴ $\boxed{k = -1}$
3)	In ΔADE and ΔCBE, ∠AED ≅ $\boxed{\angle CEB}$ ...[Common angle] ∠DAE ≅ ∠BCE ...$\boxed{\text{[Angles inscribed in the same arc]}}$ ∴ ΔADE ~ $\boxed{\Delta CBE}$ ...$\boxed{\text{[AA test of similarity]}}$ ∴ $\frac{AE}{\boxed{CE}} = \frac{\boxed{ED}}{\boxed{EB}}$ ...[Corresponding sides of similar triangles] ∴ $\boxed{AE \times EB}$ = CE × ED
Q.2.B	**Solve any TWO of the following:**
1)	Suppose ☐ ABCD is a rectangle in which BC = 16 cm. Area of ☐ ABCD = AB × BC ∴ 192 = AB × 16 ∴ AB = $\frac{192}{16}$ = 12 cm Now, in ΔABC, ∠B = 90° ………. [Angle of a rectangle] ∴ $AC^2 = AB^2 + BC^2$ …….. [Pythagoras theorem] $= 12^2 + 16^2$ = 144 + 256 = 400 ∴ AC = $\sqrt{400}$ ………. [Taking square root of both sides] = 20 cm **∴ The diagonal of the rectangle is 20 cm.**

2)





Rough figure

Steps of construction:

i) Draw a circle of radius 3.6 cm and take any point K on it.

ii) Draw chord BK of any length and an inscribed ∠BAK of any measure.

iii) By taking A as centre and any convenient distance on compass draw an arc intersecting the arms of ∠BAK in points P and Q.

iv) With K as centre and the same distance in the compass, draw an arc intersecting the chord BK at point S.

v) Taking radius equal to PQ and S as centre, draw an arc intersecting the previously drawn arc. Name the point of intersecting as R.

vi) Draw line RK.

Line RK is the required tangent to the circle.

3)

Given: Radius (R) = 6 cm, area of sector = $15\pi\ cm^2$

To find: i) Measure of the arc (θ), ii) Length of the arc (l)

Area of sector = $\frac{\theta}{360} \times \pi r^2$

$\therefore 15\pi = \frac{\theta}{360} \times \pi \times 6^2$

$\therefore 15\pi = \frac{\theta}{360} \times \pi \times 36$

$\therefore 15 = \frac{\theta}{10}$

$\therefore \theta = 150°$

Also. area of sector = $\frac{l \times r}{2}$

$\therefore 15\pi = \frac{l \times 6}{2}$

$\therefore l = \frac{15\pi \times 2}{6} = 5\pi$ cm

∴ **The measure of the arc and the length of the arc are 150° and 5π cm respectively.**

Q.3 Solve any THREE of the following:

1)

i) MT = 9 cm …………… [Radius of the bigger circle]

ii) MT = MN + NT ………. [M – N – T]

∴ 9 = MN + 2.5

∴ MN = 9 – 2.5

∴ **MN = 6.5 cm**

iii) seg MR is a tangent to the smaller circle and NS is its radius.

∴ ∠NSM = 90° ………. [Tangent theorem]

iv) In ΔNSM, ∠NSM = 90°

$\therefore MN^2 = NS^2 + MS^2$ ……..... [Pythagoras theorem]

$\therefore 6.5^2 = 2.5^2 + MS^2$

$\therefore MS^2 = 6.5^2 - 2.5^2$

= (6.5 + 2.5) (6.5 – 2.5) ………… [$\therefore a^2 - b^2 -$ = (a + b) (a – b)]

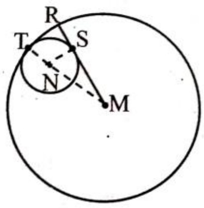
= 9 × 4 = 36

$\therefore MS = \sqrt{36}$ …………… [Taking square root of both sides]

= 6 cm

But, MR = MS + SR ……….. [M – S – R)

∴ 9 = 6 + SR

	$\therefore$ SR = 9 – 6 $\therefore$ SR = 3 cm Now, $\frac{MS}{SR} = \frac{6}{3} = \frac{2}{1}$ $\therefore$ $\mathbf{\frac{MS}{SR} = 2 : 1}$ 
2)	**Given:** For the conical water jug, radius (r) = 3.5 cm, height (h) = 10 cm For the cylindrical water pot, Radius (R) = 7 cm, height (H) = 10 cm **To find:** Number of jugs of water the cylindrical pot can hold. Volume of conical jug = $\frac{1}{3}\pi r^2 h$ $= \frac{1}{3} \times \pi \times 3.5^2 \times 10$ $= \frac{1}{3} \times 3.5^2 \times 10\pi$ cm^3 Volume of cylindrical pot = $\pi R^2 H$ $= \pi \times 7^2 \times 10$ $= 49 \times 10\pi$ cm^3 Number of jugs = $\frac{\text{Volume of cylindrical pot}}{\text{Volume of conical jug}}$ $= \frac{49\times10\pi}{\frac{1}{3}\times3.5^2\times10\pi} = \frac{49\times3}{3.5\times3.5}$ $= \frac{49\times3\times100}{35\times35} = 12$ $\therefore$ **The cylindrical pot can hold 12 jugs of water.**
3)	Let A(x_1, y_1), B(x_2, y_2) and P (x, y) be the given points. Here, $x_1 = 8$, $y_1 = 9$, $x_2 = 1$, $y_2 = 2$, x = k, y = 7 $\therefore$ By section formula, $y = \frac{my_2+ny_1}{m+n}$ $\therefore 7 = \frac{2m+9n}{m+n}$ $\therefore$ 7m + 7n = 2m + 9n $\therefore$ 5m = 2n $\therefore \frac{m}{n} = \frac{2}{5}$ $\therefore$ m = 2, n = 5 $x = \frac{mx_2+nx_1}{m+n}$ $\therefore k = \frac{2(1)+5(8)}{2+5} = \frac{2+40}{7} = \frac{42}{7} = 6$ $\therefore$ **Point P divides seg AB in the ratio 2 : 5, and the value of k is 6.**
4)	**Proof:** Diagonals AC and BD intersect at point M. ………… [Given] $\therefore$ M is the midpoint of diagonals AC and BD. ……..[Diagonals of a parallelogram bisect each other] $\therefore$ AM = $\frac{1}{2}$ AC, and MD = $\frac{1}{2}$ BD …….. (i) In ΔABD, M is the midpoint of BD. $\therefore AB^2 + AD^2 = 2\,AM^2 + 2\,MD^2$ …….. [Apollonius theorem] $\therefore AB^2 + AD^2 = 2\left(\frac{1}{2}AC\right)^2 + 2\left(\frac{1}{2}BD\right)^2$ …….. [From (i)] $\therefore AB^2 + AD^2 = \frac{1}{2}AC^2 + \frac{1}{2}BD^2$

$\therefore 2AB^2 + 2AD^2 = AC^2 + BD^2$ ……….[Multiplying both sides by 2]

$\therefore AB^2 + AB^2 + AD^2 + AD^2 = AC^2 + BD^2$ ……(ii)

But, AB = CD and BC = AD ………..[Opposite sides of a parallelogram]

$\therefore \mathbf{AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + AD^2}$ …….[From (ii) and (iii)]

Q.4 Solve any ONE of the following

1) Let 1 + sin x = m

L.H.S. = $\frac{1+\sin x-\cos x}{1+\sin x+\cos x} + \frac{1+\sin x+\cos x}{1+\sin x-\cos x}$

$= \frac{m-\cos x}{m+\cos x} + \frac{m+\cos x}{m-\cos x}$

$= \frac{(m-\cos x)^2+(m+\cos x)^2}{(m+\cos x)(m-\cos x)}$

$= \frac{m^2-2m\cos x+\cos^2 x+m^2+2m\cos x+\cos^2 x}{m^2-\cos^2 x}$

$= \frac{2m^2+2\cos^2 x}{m^2-\cos^2 x} = \frac{2(m^2+\cos^2 x)}{(m^2-\cos^2 x)}$

$= \frac{2[(1+\sin x)^2+\cos^2 x]}{(1+\sin x)^2-\cos^2 x} = \frac{2(1+2\sin x+\sin^2 x+\cos^2 x)}{1+2\sin x+\sin^2 x-\cos^2 x}$

$= \frac{2(1+2\sin x+1)}{1-\cos^2 x+2\sin x+\sin^2 x} = \frac{2(2+2\sin x)}{\sin^2 x+2\sin x \sin^2 x}$

$= \frac{2\times 2(1+\sin x)}{2\sin x+2\sin^2 x} = \frac{2\times 2(1+\sin x)}{2\sin x(1+\sin x)}$

$= \frac{2}{\sin x}$

= 2 cosec x = R.H.S.

$\therefore \mathbf{\frac{1+\sin x-\cos x}{1+\sin x+\cos x} + \frac{1+\sin x+\cos x}{1+\sin x-\cos x} = 2\ cosec\ x}$

2) **Construction:** Draw seg AD.

In ΔADB,

∠ADB = 90° (i) [Angle inscribed in a semicircle]

∴ seg AD ⊥ side BC

In ΔABC, [Given]

∠BAC = 90° and

Seg AD ⊥ hypotenuse BC [From (i)]

In ΔADB and ΔCAB,

∠ADB ≅ ∠BAC [Each 90°]

∠ABD ≅ ∠ABC [Common angle]

∴ΔADB ~ ΔCAB [AA Test of similarly]

∴ ∠BAD ≅ ∠BCA (ii) [c.a.s.t.]

∠BAD ≅ ∠HDB (iii) [Both are equal to $\frac{1}{2}$ m (arc BD)]

∠HDB ≅ ∠JDC (iv) [Vertically opposite angles]

∴ ∠JDC ≅ ∠BCA (v) [From (ii), (iii), (iv)]

In ΔJDC ≅ ∠DCJ [From (v), B-D-C]

∴ seg JD ≅ seg CJ [Converse of isosceles triangle theorem]

seg JD ≅ seg JA [Tangent segment theorem]

∴ seg AJ ≅ seg CJ

Q.5 Solve any ONE of the following

1) **Given:** Actual distance between the places A & B is 225 km.

In a map this distance is denoted by segment of length 2.5 cm.

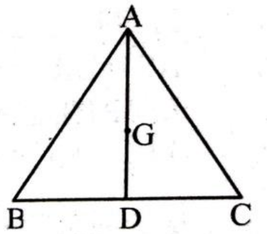
In same map distance between places A & C = 2.5 cm.

To find: actual distance between places A & C.

In given map, scale is same.

$\therefore \frac{\text{Actual distance between A \& B}}{\text{distance between A \& B in a map}} = \frac{\text{Actual distance between A \& C}}{\text{distance between A \& C in a map}}$

$\therefore \frac{225 \times 100000}{2.5} = \frac{\text{Actual distance between A \& C}}{4.2}$

	$\therefore \frac{225 \times 100000 \times 10 \times 42}{25 \times 10}$ = Actual distance between A & C $\therefore$ 378 × 100000 = Actual distance between A & C $\therefore$ Actual distance between A & C = 378 × 100000 cm = 378 km.
2)	Suppose ΔABC is an isosceles triangle. $\therefore$ AB = AC = 13 cm, BC = 10 cm AD is the median and G is the centroid. $\therefore$ D is the midpoint of side BC. $\therefore DC = \frac{1}{2} BC = \frac{1}{2} \times 10 = 5$ cm Now, $AB^2 + AC^2 = 2\ AD^2 + 2\ DC^2$ ....[Apollonius theorem] $\therefore 13^2 + 13^2 = 2\ AD^2 + 2\ (5)^2$ $\therefore 2 \times 13^2 = 2\ AD^2 + 2 \times 25$ $\therefore 169 = AD^2 + 25$ ....[Dividing both sides by 2] $\therefore AD^2 = 169 - 25$ $\therefore AD^2 = 144$ $\therefore AD = \sqrt{144} = 12$ cm We know that, the centroid divides the median in the ratio 2 : 1 $\therefore \frac{AG}{GD} = \frac{2}{1}$ $\therefore \frac{GD}{AG} = \frac{1}{2}$ ....[By invertendo] $\therefore \frac{GD+AG}{AG} = \frac{1+2}{2}$ ....[By componendo] $\therefore \frac{AD}{AG} = \frac{3}{2}$ ....[A – G – D] $\therefore \frac{12}{AG} = \frac{3}{2}$ $\therefore AG = \frac{12 \times 2}{3} = 8$ cm $\therefore$**The distance between the vertex opposite to the base and the centroid is 8 cm.** 

*This question paper is for practice purpose only.