MKCL Creating a Knowledge Lit World www.mkcl.org	**Subject – Geometry** **Std. – 10^{th} EM**	**Set – I** **Model Answer**

Q.1A. Choose the correct alternative.

1) C
2) C
3) B
4) C

Q.1.B Solve any TWO of the following:

1) **Given:** A(P-ABC) = $154\ cm^2$, radius (r) = 14 cm

i) A(P-ABC) = $\frac{\theta}{360} \times \pi r^2$

$\therefore 154 = \frac{\theta}{360} \times \frac{22}{7} \times 14^2$

$\therefore \theta = \frac{154 \times 360 \times 7}{22 \times 14^2} = 90°$

$\therefore \angle APC = 90°$

ii) l(arc ABC) = $\frac{\theta}{360} \times 2\pi r$

$= \frac{90}{360} \times 2 \times \frac{22}{7} \times 14$

$\therefore$ ***l*****(arc ABC) = 22 cm**

2) Suppose ABCD is the given quadrilateral.

$\therefore$ Slope of diagonal AC = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-7}{0-1} = \frac{-10}{-1} = 10$

Slope of diagonal BD = $\frac{3-3}{-3-6} = \frac{0}{-9} = 0$

$\therefore$ **The slopes of the diagonals of the quadrilateral are 10 and 0.**

3) Let AB represent the height of the tree, and
Point C represent the position of the boy.
BC = 60 m
Angle of elevation
= ∠ACB = 60°
In right angled ΔABC,

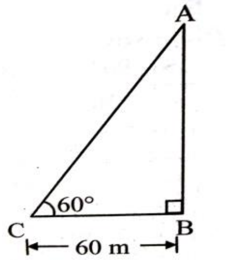
$\tan 60° = \frac{AB}{BC}$

$\therefore \sqrt{3} = \frac{AB}{60}$

$\therefore AB = 60\sqrt{3} = 60 \times 1.73$

$\therefore AB = 103.80$

$\therefore$ **The height of the tree is 103.80 m.**



Q.2A Complete the any TWO of the following activities:

1)

$\therefore$ Slope of line $\ell = \dfrac{\boxed{-1}-\boxed{3}}{\boxed{-3}-\boxed{1}} = \dfrac{\boxed{-4}}{\boxed{-4}} = \dfrac{\boxed{1}}{\boxed{1}}$

(½ mark)

Slope of line $m = \dfrac{\boxed{-2}-\boxed{3}}{\boxed{0}-\boxed{5}} = \dfrac{\boxed{-5}}{\boxed{-5}} = \dfrac{\boxed{1}}{\boxed{1}}$

Slope of line $n = \dfrac{\boxed{-1}-\boxed{1}}{\boxed{4}-\boxed{6}} = \dfrac{\boxed{-2}}{\boxed{-2}} = \dfrac{\boxed{1}}{\boxed{1}}$

From this we can verify that $\boxed{\textit{parallel}}$ lines have $\boxed{\textit{equal}}$ slopes.

2)

In ΔLMN, MN is the base and LP is the height.

In ΔDMN, MN is the base and DQ is the height.

$\therefore \quad \dfrac{A(\Delta LMN)}{A(\Delta DMN)} = \dfrac{\boxed{\mathbf{MN}} \times \boxed{\mathbf{LP}}}{\boxed{\mathbf{MN}} \times \boxed{\mathbf{DQ}}}$

...[The ratio of areas of two triangles is equal to the ratio of the product of their bases and corresponding heights]

$\therefore \quad \dfrac{A(\Delta LMN)}{A(\Delta DMN)} = \dfrac{\boxed{\mathbf{LP}}}{\boxed{\mathbf{DQ}}}$

3)

Let the radius of the circle be r.

line l is the tangent to the circle and seg OP is the radius. ...[Given]

$\therefore$ seg $\boxed{\mathbf{OP}} \perp$ line l ...[Tangent theorem]

chord RS || line l ...[Given]

$\therefore$ seg OP $\perp \boxed{\textbf{chord RS}}$

$\therefore \quad QS = \dfrac{1}{2}\ \boxed{\mathbf{RS}}$ ...[Perpendicular drawn from the centre of the circle to the chord bisects the chord]

$= \dfrac{1}{2} \times 12$

$= 6$ cm

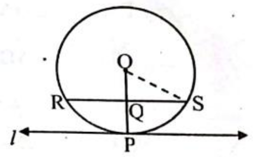
Also, $\boxed{OQ} = \frac{1}{2}$ OP ...[Q is the midpoint of OP]

$= \frac{1}{2}$ r

In ΔOQS, $\angle OQS = 90°$ ...[seg OP ⊥ chord RS]

$\therefore$ $OS^2 = \boxed{OQ^2} + QS^2$ ... **Pythagoras theorem**

$\therefore$ $r^2 = \boxed{\left(\frac{1}{2}r\right)^2} + 6^2$



$\therefore$ $r^2 = \frac{1}{4}r^2 + 36$

$\therefore$ $r^2 - \frac{1}{4}r^2 = 36$

$\therefore$ $\frac{3}{4}r^2 = 36$

$\therefore$ $r^2 = \frac{36 \times 4}{3}$

$\therefore$ $r^2 = \boxed{48}$

$\therefore$ $r = \sqrt{48}$ ...[Taking square root of both sides]

$= \boxed{4\sqrt{3} \text{ cm}}$

$\therefore$ The radius of the given circle is $\boxed{4\sqrt{3} \text{ cm.}}$

Q.2.B Solve any TWO of the following:

1) In ΔPQR, point S is the midpoint of side QR. ....[Given]

$\therefore PQ^2 + PR^2 = 2\ PS^2 + 2\ SR^2$ ....[Apollonius theorem]

$\therefore 11^2 + 17^2 = 2\ (13)^2 + 2\ SR^2$

$\therefore 121 + 289 = 2\ (169) + 2\ SR^2$

$\therefore 410 = 338 + 2\ SR^2$

$\therefore 2\ SR^2 = 410 - 338$

$\therefore 2\ SR^2 = 72$

$\therefore SR^2 = \frac{72}{2} = 36$

$\therefore SR = \sqrt{36}$ ....[Taking square root of both sides]

$= 6$ units

Now, $QR = 2\ SR$ .... [S is the midpoint of QR]

$= 2 \times 6$

$\therefore$ **QR = 12 units**

2) **Given :** Radius (r) = 10 cm, Measure of the arc (θ) = 54°

To find: i) Area of the sector, ii) length of arc

Area of sector = $\frac{\theta}{360} \times \pi r^2$

$= \frac{54}{360} \times 3.14 \times (10)^2 = \frac{3}{20} \times 3.14 \times 100$

$= 3 \times 3.14 \times 5 = 47.1\ cm^2$

Area of sector = $\frac{l \times r}{2}$

$\therefore 47.1 = \frac{l \times 10}{2}$

$\therefore l = \frac{47.1 \times 2}{10} = \frac{94.2}{10} = 9.42$ cm

$\therefore$ **The area of the sector and length of arc is 47.1 cm^2 and 9.42 cm respectively.**

3) □ MRPN is a cyclic quadrilateral.

$\therefore \angle R + \angle N = 180°$ .....[Theorem of cyclic quadrilateral]

$\therefore 5x - 13 + 4x + 4 = 180$

$\therefore 9x - 9 = 180$

$\therefore 9x = 189$

$\therefore x = \frac{189}{9}$

$\therefore x = 21$

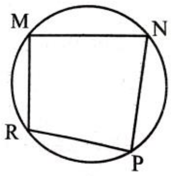
$\therefore \angle R = 5x - 13$

$= 5 \times 21 - 13 = 105 - 13 = 92°$

$\angle N = 4x + 4$

$= 4 \times 21 + 4 = 84 + 4 = 88°$

∴ m∠R = 92° and m∠N = 88°



Q.3 Solve any THREE of the following:

1) **Given:** For the conical part,

Height (h) = 4 cm, radius (r) = 3 cm

For the cylindrical part,

Height (H) = 40 cm, radius (r) = 3 cm

For the hemispherical part,

Radius (r) = 3 cm

To find: Total area of the toy.

Slant height of cone (l) = $\sqrt{h^2 + r^2} = \sqrt{4^2 + 3^2}$

$= \sqrt{16 + 9} = \sqrt{25} = 5$ cm

∴ Curved surface area of cone = $\pi r l$

$= \pi \times 3 \times 5 = 15\pi$ cm^2

Curved surface area of cylinder = $2\pi rH$

$= 2 \times \pi \times 3 \times 40 = 240\pi$ cm^2

Curved surface area of hemisphere = $2\pi r^2$

$= 2 \times \pi \times 3^2 = 18\pi$ cm^2

Total area of the toy

= Curved surface area of cone + Curved surface area of cylinder + Curved surface area of hemisphere

= $15\pi + 240\pi + 18\pi$

= 273π cm^2

∴ The total area of the toy is 273π cm^2.

2) Let The va;lue of BY be x.

BC = BY + YC ……[B –Y–C]

∴ 6 = x + YC

∴ YC = 6 – x

In ΔBAY, ray BX bisects ∠B. …….[Given]

$\therefore \frac{AB}{BY} = \frac{AX}{XY}$ ……..[Angle bisector theorem]

$\therefore \frac{5}{x} = \frac{AX}{XY}$ ……..(i)

Also, in ΔCAY, ray CX bisects ∠C……. [Given]

$\therefore \frac{AC}{YC} = \frac{AX}{XY}$ …….[Angle bisector theorem]

$\therefore \frac{4}{6-x} = \frac{AX}{XY}$

$\therefore \frac{4}{6-x} = \frac{5}{x}$ …….[From (i)]

$\therefore 4x = 5(6 - x)$

$\therefore 4x = 30 - 5x$

$\therefore 9x = 30$

$\therefore x = \frac{30}{9} = \frac{10}{3}$

Now, $\frac{AX}{XY} = \frac{5}{\left(\frac{10}{3}\right)}$

$= \frac{5 \times 3}{10}$ ……..[Substituting the value of x in equation (i)]

$\therefore \mathbf{\frac{AX}{XY} = \frac{3}{2}}$

3) L.H.S. $= \frac{\tan\theta}{\sec\theta - 1}$

$= \frac{\tan\theta}{\sec\theta-1} \times \frac{\sec\theta+1}{\sec\theta+1}$ ………[On rationalising the denominator]

$= \frac{\tan\theta(\sec\theta+1)}{\sec^2\theta-1}$

$= \frac{\tan\theta(\sec\theta+1)}{\tan^2\theta}$ ……..[$\therefore \sec^2\theta - 1 = \tan^2\theta$]

$= \frac{\sec\theta+1}{\tan\theta}$

Now, $\frac{\tan\theta}{\sec\theta-1} = \frac{\sec\theta+1}{\tan\theta}$

$\therefore$ By theorem on equal ratios,

$\frac{\tan\theta}{\sec\theta-1} = \frac{\sec\theta+1}{\tan\theta} = \frac{\tan\theta+(\sec\theta+1)}{\sec\theta-1+(\tan\theta)}$

$= \frac{\tan\theta+\sec\theta+1}{\tan\theta+\sec\theta-1}$

= R.H.S.

$\therefore \boldsymbol{\frac{\tan\theta}{\sec\theta-1} = \frac{\tan\theta+\sec\theta+1}{\tan\theta+\sec\theta-1}}$

4) **Given:** □ ABCD is a parallelogram. M is the midpoint of side CD.

To prove: EL = 2BL

i) □ AQST is a cyclic quadrilateral ………[Given]

$\therefore$ ∠TAQ + ∠TSQ = 180° [Theorem of cyclic quadrilateral]

ii) line PR is the tangent and seg AQ is the secant ……[Given]

$\therefore$ ∠AQP = $\frac{1}{2}$ m(arc AQ) ……….[Theorem of angle between tangent and secant]

But, ∠ASQ = $\frac{1}{2}$ m(arc AQ) ……[Inscribed angle theorem]

$\therefore$ ∠AQP ≅ ∠ASQ

Similarly, we can prove that,

∠AQP ≅ ∠ATQ

iii) ∠QTS = $\frac{1}{2}$ m(arc QS) ……..[Inscribed angle theorem]

But, ∠SQR = $\frac{1}{2}$ m(arc QS) ……[Theorem of angle between tangent and secant]

$\therefore$ ∠QTS ≅ ∠SQR

Also, ∠QTS = ∠QAS …….[Angles inscribed in the same arc]

iv) ∠TQS = ∠TAS …….[Angles inscribed in the same arc]

$\therefore$ ∠TQS = 65°

Now, ∠TQS = $\frac{1}{2}$ m(arc TS) …..[Inscribed angle theorem]

$\therefore$ 65° = $\frac{1}{2}$ m(arc TS)

$\therefore$ m(arc TS) = 65° × 2

$\therefore$ **m(arc TS) = 130°**

Q.4 Solve any ONE of the following

1) □ABCD is a parallelogram [Given]

side AE ∥ side BC on transversal BE

∠AEB ≅ ∠CBE (i) [Alternate angles]

In ΔALE and ΔCLB,

∠AEL ≅ ∠CBL [From (i) and B–L–E]

∠ALE ≅ ∠CLB [Vertically opposite angles]

$\therefore$ ΔALE ~ ΔCLB [By A–A test of similarity]

$\therefore \frac{EL}{BL} = \frac{AE}{BC}$ (ii) [Corresponding sides of similar triangles]

In ΔDME and ΔBMC,

Seg DM ≅ seg CM [M is midpoint of DC]

∠DME ≅ ∠CMB [Vertically opposite angles]

∠DEM ≅ ∠CBM [From (i), B–M–E, A–D–E]

$\therefore$ ΔDME ≅ ΔBMC [By S A A test of congruency]

$\therefore$ DE = BC (iii) [c.s.c.t.]

Also, AD = BC (iv) [Opposite sides of parallelogram]

Now, $AE = AD + DE$ (v) [A–D–E]

$\therefore AE = BC + BC$ [From (iii), (iv) and (v)]

$\therefore AE = 2BC$

$\therefore \frac{AE}{BC} = \frac{2}{1}$ (vi)

$\therefore \frac{EL}{BL} = \frac{2}{1}$ [From (ii) and (vi)]

2) Consider, $2ab = 2 \times A(\Delta PQR) \times QR$

$= 2 \times (\frac{1}{2} \times PR \times QN) \times QR$

$\therefore 2ab = PR \times QN \times QR$ (i)

Consider, $b^4 + 4a^2$

$= QR^4 + 4[A(\Delta PQR)]^2$

$= QR^4 + 4\left[\frac{1}{2} \times PQ \times QR\right]^2$

$= QR^4 + 4 \times \frac{1}{4} \times PQ^2 \times QR^2$

$= QR^4 + PQ^2 \times QR^2$

$\therefore b^4 + 4a^2 = QR^2\ [QR^2 + PQ^2]$

$\therefore \frac{b^4+4a^2}{QR^2} = QR^2 + PQ^2$ (ii)

In ΔPQR,

$\angle PQR = 90°$

$\therefore QR^2 + PQ^2 = PR^2$ (iii) [By Pythagoras theorem]

$\therefore b^4 + 4a^2 = QR^2 \times PR^2$ [From (ii) and (iii)]

$\therefore \sqrt{b^4 + 4a^2} = \sqrt{QR^2 \times PR^2}$ [Taking square root on both sides]

$\therefore \sqrt{b^4 + 4a^2} = QR \times PR$ (iv)

$\therefore \frac{2ab}{\sqrt{b^4+4a^2}} = \frac{PR \times QN \times QR}{QR \times PR}$ [Dividing (i) by (iv)]

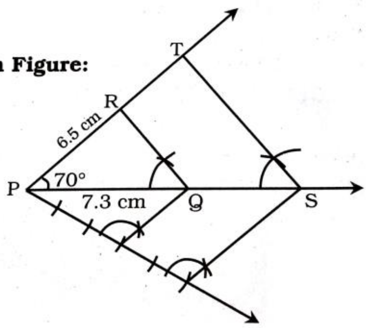
$\therefore \mathbf{QN} = \frac{\mathbf{2ab}}{\sqrt{\mathbf{b^4+4a^2}}}$

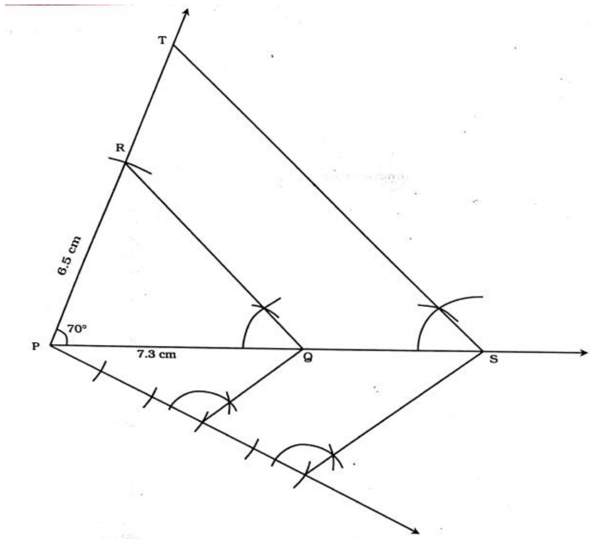
Q.5 Solve any ONE of the follwing:

1) **Given:** $\Delta PQR \sim \Delta PST$ such that $\frac{PR}{PT} = \frac{3}{5}$

$PQ = 7.3$ cm and $PR = 6.5$ cm and $\angle RPQ = 70°$ Construct ΔPQR and then construct ΔPST by dividing seg PQ in 3 equal parts and then finding out position of point S on line PQ such that P – Q – S and $\frac{PQ}{PS} = \frac{3}{5}$

Rough Figure:





2) **Given:** $\sin (A + B + C) = 1$

$\tan (A - B) = \frac{1}{\sqrt{3}}$

$\cos (A + C) = \frac{1}{2}$

To find: value of A, B & C

$\sin (A + B + C) = 1$ (given)

We know that

$\sin 90° = 1$

$\therefore \sin (A + B + C) = \sin 90°$

$\therefore A + B + C = 90°$ …….(i)

$\tan (A - B) = \frac{1}{\sqrt{3}}$ ……(given)

We know that

$\tan 30° = \frac{1}{\sqrt{3}}$

$\tan (A - B) = \tan 30°$

$A - B = 30°$ ……..(ii)

$\cos (A + C) = \frac{1}{2}$ …….(given)

We know that

$\cos 60° = \frac{1}{2}$

$\therefore A + C = 60°$ ………(iii)

From (ii) we get

$B = A - 30°$

From (iii) we get

$C = 60° - A$

Substituting these values in equation (i), we get

$A + A - 30° + 60° - A = 90°$

$A + 30° = 90°$

$A = 90° - 30°$

$A = 60°$

From (ii) we get

$B = A - 30° = 60° - 30° = 30°$

$\therefore$ $\boxed{\mathbf{B = 30°}}$

From (iii) we get

$C = 60° - A = 60° - 60° = 0°$

$\boxed{\mathbf{C = 0°}}$